

Physical measurement

* Measurement

Comparing unknown physical quantity to known one (called the unit of measurement) to know how many times the first containment on the second

* The importance of measurement

Change our observations to amount can be expressed by the numbers, such as a person temperature's degree is high (*inaccurate expression*) but a person temperature's degree is 40°C (*accurate expression*)

* Examples of physical quantities:

Everything that can be measured is called the *physical quantity* such as length, weight, blood pressure, the rate of heart beats, temperature, the level of hemoglobin in the blood, Cholesterol, mass, time and volume

* Measurement requirements:

- 1- *Physical quantities to be measured:* such as the distance between two countries or the mass of the body
- 2- *Measurement tools needed:* such as standard meter and common balance
- 3- *Units of measurement used:* standard units such as meter or kilogram

Classification of physical quantities

Basic physical quantities	Derivable physical quantities
Quantities which can't be derived from other physical quantities such as <i>length, mass, time, temperature and electric charge</i>	Quantities which can be derived from other physical quantities such as <i>speed, volume, acceleration, work, power, energy and force</i>

Example:

Volume is a derivable physical quantity which is derived from length

Volume of cuboid = length × width × height

$$V_{ol} = L \times L \times L = L^3$$

**** Systems that define the basic physical quantities and its own measuring units**

Measuring units			
Basic quantity	Measuring systems		
	French system (Gauss) C.G.S	British system (F.P.S)	Modern metric system (International) (M.K.S)
Length	Centimeter (cm)	Feet	Meter
Mass	Gram (gm)	Pound	Kilogram (Kg)
Time	Second (s)	Second (s)	Second (s)

***Mathematical equation:**

A concise form of the physical characterization long to explain in words

Note that:

Physical quantities can be expressed and their relationship to each other by mathematical equations (*Integration of physics with mathematics*)

***Properties of mathematical equation:**

For each equation meaning called the physical sense (meaning)

International system of units	
Physical quantity	Unit
Length (L)	Meter (m)
Mass (m)	Kilogram (kg)
Time (T)	Second (s)
Electric current intensity (I)	Ampere (A)
Absolute temperature (T)	Kelvin (K)
Amount of matter (n)	Mole (mol)
Luminosity (I_v)	Candela (cd)

**** Two additional units has been added:**

1- **Radian** for planer (flat) angle

2- **Astrdian** for stereophonic angle

Measuring tools

Some old and recent measurement tools

Length scale	Metric tape, ruler, micrometer and Vernier caliper
Mass scale	Roman balance, two pan balance, one pan balance and digital balance
Time scale	Hourglass, pendulum clock, stopwatch and digital clock

Standard units

It is a measuring units internationally agreed and used in the International System

Units

Note that:



Any physical quantity defined by two factors (*number and unit*)

We can't mention just number without determine the unit, which is measured by that amount such as *standard length is standard meter, standard mass is standard kilogram and standard time is second*

*Standard meter:

It is the distance between the two marks engraved at both ends of rod of the alloy platinum – Iridium kept at zero °C at the International Bureau of measurements and scales near Paris

*Standard kilogram:

Equal to the cylinder block from the alloy platinum – Iridium with specific dimensions kept at zero °C at the International Bureau of measurements and scales near Paris

*Second:

The second is defined as equal to $\frac{1}{86400}$ the average solar day.

Note that:

Day = 24 hours = 24 × 60 minute = 24 × 60 × 60 second = 86400 second

Scientists have suggested that the use of atomic clocks such as *cesium clock* to measure time, it is extremely accurate.



Dimensional equation (formula)

In the equation uses three basic symbols dimensions; *length (L), mass (M) and time (T)*

1- Dimensional equation used in the expression of most of the derivable physical quantities in terms of dimensions basic physical *quantities (length, mass and time upped for a certain power)* and written expression as the following

$$[A] = L^a M^b T^c$$

Where $[A]$ is the physical quantity, (a, b and c) are dimensions (power) L, M and T respectively $Speed = \frac{distance}{time} = \frac{Length}{time}$ So dimensional equation for the speed is

$$V = \frac{L}{T} = L T^{-1}$$

2- Dimensional equation used to determine a unit of measurement derivable physical quantities for example, from the dimensional equation of the speed ($V = L T^{-1}$) so the measuring unit is ($m s^{-1}$ or m/s)

3- We can add or subtract two physical quantities provided that

- a- Should be the same type and the same dimensional equation
- b- Should be the same measuring unit.

Note that:

You cannot add a body of mass 2 kg to a distance of 2 meters

If a different unit of measurement for the two quantities of the same type, so you should change a unit of measurement one of them to the unit of measurement of the second to add or subtracting the two quantities with each other.

$$1 m + 170 cm = 100 cm + 170 cm = 270 cm$$

4- If we multiply or divide two physical quantities have the same dimensional equation, we get a new physical quantity (*dividing the velocity on the time we get the acceleration*)

5- Numbers, fractions and numerical constants such as ($\pi = \frac{22}{7}$) don't have dimensions

* Importance of dimensional equation:

Dimensional equation can be used to test the validity of laws where the dimensions must be both sides of the equation are identical, and this so-called investigation dimensional homogeneity of the equation

Example:

If you know that acceleration *is the rate of change of speed (velocity)*, find dimensional equation and measuring unit

$$\text{acceleration} = \frac{\text{velocity}}{\text{time}} \therefore a = \frac{v}{t} = \frac{d}{t \times t} = \frac{L}{T^2} = L T^{-2}$$

Because the dimensional equation of the acceleration is $L T^{-2}$, so the measuring unit is $m s^{-2}$ or m / s^2

Physical quantity	Relationship with other quantities	Dimensional equation	Measuring unit
Area (A)	length \times width	$L \times L = L^2$	m^2
Volume (V_{ol})	length \times width \times height	$L \times L \times L = L^3$	m^3
Density (ρ)	mass \div volume ($\rho = \frac{M}{V_{ol}}$)	$M/L^3 = ML^{-3}$	kg / m^3
Force (F)	mass \times acceleration ($F = m \times a$)	$M \times L T^{-2}$ $= M L T^{-2}$	$kgm/s^2 = N$
Work, energy (W)	force \times displacement $W = F \times d$	$M L T^{-2} \times L$ $= M L^2 T^{-2}$	kgm^2/s^2 $= N.m = J$
Power (P)	work \div time $P = \frac{w}{t}$	$\frac{M L^2 T^{-2}}{T}$ $= M L^2 T^{-3}$	kgm^2/s^3 $= N.m/s$ $= J/s = w$

Example:

Proved true, the following relationship ($K.E = \frac{1}{2} m \times V^2$), if you know the dimensions equation of energy is ($E = M L^2 T^{-2}$)

Solution:

Dimensional equation for the left side is $M L^2 T^{-2}$

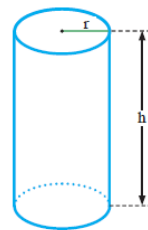
It is known that the fraction $\frac{1}{2}$ has no measuring unit

Dimensional equation for the right side $M L^2 / T^2 = M L^2 T^{-2}$

And it is the same dimensional equation of the left side

Example:

Someone suggested that the volume of the cylinder determines from the relationship $V_{ol} = \pi r h$, where (r) is the radius of the cylinder base and (h) is cylinder height, prove by using dimensional equation the validity of that equation

**Solution:**

Dimensional equation for the left side is L^3 (volume)

Dimensional equation for the right side $L \times L = L^2$

It is known that the constant (π) has no measuring unit

Dimensions sides of the equation do not match, so equation error

Note that:

Having the same dimensional equation on both sides of the equation does not guarantee the correctness, but differ on both sides of the equation confirms mistake

***Problem:**

The movement of the body under the influence of gravity as the following relationship ($v_t = v_0 + g t$), prove the validity of this relationship by using dimensional equation. (g) gravity, (t) time, (v_t) terminal velocity and (v_0) initial velocity

***Multiplying and fractions of units in international (global) system**

Prefers the expression of very large numbers and using a very small number 10 is raised for a particular exponent (*power of tenth*)

This method is called in the expression of physical quantities as *standard formula to write numbers*

For example, if the distance between the stars is estimated *100,000,000,000,000,000 m*, writes as *standard writing numbers = $1 \times 10^{17} m$*

If the distance between the atoms of solids about *0.000000001 m*, writes as *standard writing numbers = $1 \times 10^{-9} m$*

Factor 10^{\pm} called specific names, scientists agreed upon a convertible examples

Factor	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^3	10^6	10^9
Name	Nano	Micro	Millie	Centi	Kilo	Mega	Giga
Symbol	n	μ	m	c	K	M	G

From bigger to smaller (times), from smaller to bigger (divide)

***Problem (1):** the volume of water in the tank = 9m^3 , calculate the volume of water in cm^3 , mm^3 , km^3 , μm^3 and Mm^3 units

***Solution:**

The volume of water in the reservoir = $9 \times (10^2 \times 10^2 \times 10^2) = 9 \times 10^6 \text{ cm}^3$

The volume = $9 \times (10^3 \times 10^3 \times 10^3) = 9 \times 10^9 \text{ mm}^3$

The volume = $9 \times (10^{-3} \times 10^{-3} \times 10^{-3}) = 9 \times 10^{-9} \text{ km}^3$

The volume = $9 \times (10^6 \times 10^6 \times 10^6) = 9 \times 10^{18} \mu\text{m}^3$

The volume = $9 \times (10^{-6} \times 10^{-6} \times 10^{-6}) = 9 \times 10^{-18} \text{ Mm}^3$

*** Problem (2):** electrical current intensity is 7 m.A express the intensity of electric current in microampere unit

***Solution:** $1\text{m.A} = 10^{-3} \text{ A} \therefore 1 \mu\text{.A} = 10^{-6} \text{ A} \therefore \frac{1\text{m.A}}{1\mu\text{.A}} = 10^3 \therefore 1\text{mA} = 10^3 \mu\text{A}$

$\therefore 7\text{mA} = 7 \times 10^3 \mu\text{A}$, this means that $7 \text{ mA} = 7000 \mu\text{A}$

*** Measurement error:**

Measurement process cannot be accurately 100%, *but there should be even a simple percentage of error.*

***Causes an error in the measurement:**

1-Choosing measuring instrument not suitable *such as using*

the common balance rather than the sensitive balance in estimating the mass of gold ring

2-The existence of a defect in the measurement tool, *example defects ammeter*

a-That the device is out of date and magnet inside became weak

b-Exit pointer ammeter zero staging when you disconnect the electric current

3- A measurement error manner *such as*

a- The lack of knowledge of the use of multiple devices staging such as Maltmiter.

b- Consideration to the pointer at an angle rather than a vertical line of sight on the tool

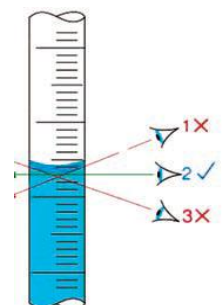
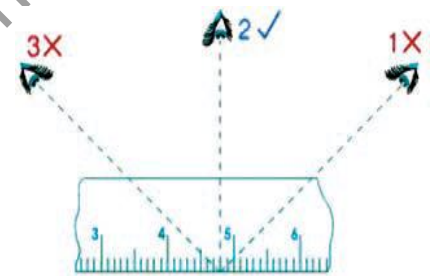
4- Environmental factors (*temperature, humidity and air currents*)

****Types of measurement:**

1-Direct measurement (using a single tool as a measure of the density by using the *hydrometer*)

2- Indirect measurement (using more than one tool to measure density of

the liquid by measuring the mass using balance, as well as volume by using measuring cylinder and then dividing the mass over volume)



***Comparison between the measurement of direct and indirect**

P. O. C	Direct measurement	Indirect measurement
The number of measurement	One measuring process	More than one measuring process
Calculations	Do not be substituted in a mathematical process	Be substituted in a mathematical relationship to calculate the quantity
Errors in measurement	There is one error in the measurement process	There may be several errors in the measurement process (<i>it happens what is known as the accumulation of errors</i>)
Example	Measuring volume using a graduated cylinder	Measure the volume by measuring the length, width and height

1- Calculation error in the case of direct measurement:**a- Absolute error** (Δx):

It is the difference between the true (x_0) value and the measured value (x)

$\Delta x = |x_0 - x|$, mark scale | | indicates that the product is always positive even if it were real quantity less than the measured quantity because what is important is to know how much error whether it increases or decreases *for instance*, $|d - 8d| = 8$

b- Relative error (r):

It is the ratio between the absolute error (Δx) to the true value (x_0) $r = \frac{\Delta x}{x_0}$

***Problem (1):**

One of the students measured the length of a pencil and found it equals (9.9 cm) and the real value of a pencil length is equal to (10 cm), while classmate measured the length of the class room he found it equals (9.13 m), while the real value of the length of classroom (9.11m). *Calculate the absolute error and relative error in each case*

Solution:**In case of first student:**

- *Calculation the absolute error* $\Delta x = |x_0 - x| = |10 - 9.9| = 0.1 \text{ cm}$

- *Calculation the relative error* $r = \frac{\Delta x}{x_0} = \frac{0.1}{10} = 0.01 = 1\%$

***In case of second student:**

- **Calculation the absolute error:** $\Delta x = |x_0 - x| = |9.11 - 9.13| = |-0.02| \text{ m} = 2 \text{ cm}$

- **Calculation the relative error:** $r = \frac{\Delta x}{x_0} = \frac{0.02}{9.11} = 0.0022 = 0.22\%$

- **And can be expressed as a result of the measurement process as follows**

Length of a pencil = $(10 \pm 0.1) \text{ cm}$ & Length of classroom = $(9.11 \pm 0.02) \text{ m}$

Note that

The absolute error in the measurement of the length of the classroom is larger than the absolute error in the measurement of the length of a pencil, however we find that the relative error in the measurement of the length of classroom is less, **and this indicates that measure the length of classroom more accurate than measuring the length of a pencil**

The relative error of more significance in the accuracy of the measurement than the absolute error, and be **more accurate measurement whenever the relative error is small**

2- Calculation error in the indirect measurement

The method of calculating the error in the case of indirect measurement vary depending on the mathematical relationship during the calculation process

Mathematical relationship	Example	How to calculate the error
Add	Measure the volume of two quantities of liquid	Absolute error = absolute error in the first measurement + absolute error in the second measurement $\Delta x = \Delta x_1 + \Delta x_2$
Subtract	Measuring the volume of a coin subtracting the volume of water before you put them in a graduated cylinder the volume of the water after placed in the cylinder	
Times	Measure the area of a rectangle measuring the length and width and finding $(L \times W)$	Relative error = relative error in first measurement + relative error in second measurement $r = r_1 + r_2$
Divide	Measure the density of the liquid, measure mass and volume then find $\text{mass} \div \text{volume}$	

***Problem (2):**

Calculate the relative error and the absolute error in measuring the area of the rectangle (A) its length (6 ± 0.1) , and width (5 ± 0.2)

***Solution:**

Relative error in the measurement of length $r_1 = \frac{\Delta x}{x_0} = \frac{0.1}{6} = 0.017$

Relative error in the measurement of width $r_2 = \frac{\Delta x}{x_0} = \frac{0.2}{5} = 0.04$

Relative error in the measurement area $r = r_1 + r_2 = 0.017 + 0.04 = 0.057$

And that's where $r = \frac{\Delta A}{A_0} \therefore \Delta A = r \times A_0 = 0.057 \times (5 \times 6) = 1.7 \text{ m}^2$

Based on the above the area of the rectangle (A) = $(30 \pm 1.7) \text{ m}^2$

***Problem (3):**

In a practical experiment to find the amount of physical quantity (L), which produces of adding two physical quantities L_1, L_2 , if $L_1 = (5.2 \pm 0.1) \text{ cm}$ & $L_2 = (5.8 \pm 0.2) \text{ cm}$, calculate the value of (L)

***Solution:**

Calculating the real value of (L) $L_0 = (5.2 + 5.8) = 11 \text{ cm}$

Calculating the absolute error of (L) $\Delta L = (0.1 + 0.2) = 0.3 \text{ cm} \therefore L = (11 \pm 0.3) \text{ cm}$

***Problem (4):**

Calculate the relative error and absolute error in measuring the volume of the cuboid, if the results of measuring its dimensions are as follows

Dimension	Measured quantity (cm)	True quantity (cm)
Length (x)	4.3	4.4
Width (y)	3.3	3.5
Height (z)	2.8	3

Solution:**First : calculating the relative error**

Calculating the relative error in the measurement of length $r_1 = \frac{\Delta x}{x_0} = \frac{|4.4-4.3|}{4.4} = 0.023$

Calculating the relative error in the measurement of length $r_2 = \frac{\Delta y}{y_0} = \frac{|3.5-3.3|}{3.5} = 0.057$

Calculating the relative error in the measurement of height $r_3 = \frac{\Delta z}{z_0} = \frac{|3-2.8|}{3} = 0.067$

Calculating the relative error in the measurement of volume $r = r_1 + r_2 + r_3 = 0.023 + 0.057 + 0.067 = 0.147$

***Second: calculating the absolute error:**

Calculate the true volume of the cuboid: $V_0 = x_0 y_0 z_0 = 4.3 \times 3.5 \times 3 = 46.2 \text{ cm}^3$

$r = \frac{\Delta V}{V_0} \therefore \Delta V = r V_0 = 0.147 \times 46.2 = 6.79 \text{ cm}^3$